

# **Fractional Order Motion Controls**

# Fractional Order Motion Controls

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*To my father XianShu Luo and my mother Gui'E Xiong  
– Ying Luo*

*To my family, my mentors and my colleagues  
– YangQuan Chen*

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# Foreword

I am pleased that Professor Ying Luo and Professor YangQuan Chen have completed their book, *Fractional Order Motion Controls*, and it will be published by John Wiley & Sons Ltd. The mathematical backbone of fractional order control is fractional order differential equations and fractional order transfer functions. Fractional order transfer functions were proposed by Bode in the analysis and feedback amplifier design as early as in the mid-1940s. Tustin applied Bode's idea in the late 1950s to motion control, and he suggested that the open loop transfer function of the motion control system may be set  $G(s) = (\frac{w_c}{s})^k$  with  $k = 1.5$  over a certain frequency range, say,  $(0.2 \sim 1.2)w_c$ . It is easy to check that for this transfer function the phase margin is 45 degree, and the closed loop system remains robust if this phase margin can be maintained over a finite frequency range. This example may be a strong motivation for motion control engineers to perform fractional order design. In tracing references on fractional order control, I noted that a number of interesting successful applications of fractional order control have been reported, and that excellent papers were published by control practitioners in industry such as Dr. Manabe at Mitsubishi. We may wonder then why fractional order control has not been more widely used as yet. One reason may be that fractional order controllers must be approximated by a combination of standard (integer order) differentiators and integrators for implementation and that implementable controllers may become high order. This is no longer a major obstacle since high order controllers may be digitally implemented. I think that fractional order control should not be regarded as a new control theory to replace standard control such as PID control based on integer order transfer functions. Good ideas that we learned in the standard (integer order) control theory all remain as valid design guidelines. Fractional order control adds something more to what we know and practice. It may be regarded as a means that allows us to design control systems, for example perform loop shaping, in a wider design domain. When we are freed from the constraint that each element of a controller must be of integer order, we may move only in the direction to have better control systems. Thus, fractional order control is of interest for those who design controllers for physical systems:

for example motion control systems, automotive suspension systems, robots, and so on. This book by Luo and Chen covers the range from fundamental fractional calculus and early development of fractional order control to the most recent developments by the authors themselves as well as by others. It is a fun book to read, and it will surely motivate more motion control engineers to plunge into the world of fractional thinking.

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# Preface

There is increasing interest in dynamic systems and controls of non-integer orders or fractional orders. Traditional calculus is based on integer order differentiation and integration. The concept of fractional calculus has tremendous potential to change the way we model and control the world around us. Rejecting fractional derivatives is like saying that zero, fractional, or irrational numbers do not exist. Fractional calculus has a firm and enduring theoretical foundation. However, the fractional calculus concept was not widely applied in control engineering for hundreds of years, because the idea was unfamiliar and the fractional operators were limited in their realization. In the past few decades, with the rapid development of computer technology and better understanding of the potential of fractional calculus, the realization of fractional order control systems became much easier and fractional calculus is becoming more and more useful in various science and engineering areas. The present book focuses on fractional order control of motion systems.

Motion control is a sub-field of automation, in which the velocity and position of machines are controlled using certain types of actuation devices such as a hydraulic actuator, a linear actuator, or an electric motor, generally called a servo. Motion control is an important part of robotics and Computerized Numerical Control (CNC) machine tools, and is widely used in packaging, printing, textile, semiconductor production, and the assembly industries. In motion control systems, the control strategies should be stabilizing, fast and precise. In real-time applications, high performance motion control systems must be immune to any kind of disturbance. Thus, motion control research explores enhancement of both performance following command and disturbance rejection. The aim of this book is to introduce fractional calculus-based control methods in motion control applications and to illustrate the advantages and importance of using fractional order controls.

In order to improve the performance following command of motion control, fractional order PID controllers are proposed and designed in a systematic way for integer/fractional order velocity and position systems in this work. With the “flat phase” tuning constraint and other specifications, the motion control systems based on fractional calculus can achieve better robust performances with respect to loop



gain variations or time constant variations than using traditionally optimized integer order controllers. From our extensive simulation and experimental efforts, we demonstrated the desirable control performance with faster response and smaller overshoot using properly designed fractional order controllers over those using optimized integer order controllers. In terms of systematic design schemes for fractional order PID controllers satisfying the desired specifications, stability is the minimum requirement for the controller design, and it is better to obtain a feasible region to check the complete set of specifications before the controller is designed and tuned. Therefore, the complete stability regions of the fractional order PID controller parameters, and the achievable regions of the specifications to obtain stabilizing and the desired fractional order PID controllers are discussed in detail in this book. Impressively, the achievable regions of specifications using fractional order PID controllers are significantly larger than those using an integer order PID controller for certain types of systems.

Motion control systems are usually influenced by various disturbances. In high performance motion control systems, maintaining a stable and robust operation by attenuating the influence of disturbances is required. A fractional order disturbance observer (DOB) based on the fractional order  $Q$ -filter is presented. A nice feature of this is that the traditional DOB is extended to the fractional order DOB (FO-DOB) with the advantage that the FO-DOB design will no longer be conservative nor aggressive. In addition, a fractional order adaptive feedforward cancellation (FO-AFC) scheme is proposed to cancel periodic disturbances. This FO-AFC method is much more flexible than the integer order AFC in preventing periodic disturbance and suppressing the harmonics or the noise. Meanwhile, a fractional order robust control method is devised for cogging effect compensation on the permanent magnetic synchronous motor position and the velocity systems. Also presented is a fractional order periodic adaptive learning compensation method to reject general state-dependent periodic disturbances.

In this book, nonlinear motion control systems are also considered for fractional calculus applications. A fractional order PID controller design scheme is presented for a DC motor control system with an elastic shaft. Under the same optimization conditions, the best fractional order PID controller outperforms the best integer order PID controller for the motion control system with nonlinearities of backlash and dead zone. Applying the systematic design of fractional order PD (FOPD) controller for ultra-low speed position tracking with a significant nonlinear friction effect, the experimental tracking performance using the designed FOPD controller is much better than that using the optimized integer order PI controller. This advantage of the designed FOPD is explained by the describing function analysis. Furthermore, an optimized fractional order conditional integrator (OFOCI) is proposed. By tuning the fractional order and the other tuning parameter following the analytical optimal design specifications, this proposed OFOCI can achieve an optimized performance not achievable by integer order conditional integrators.

In order to further validate and demonstrate some of the presented fractional order controller design schemes in this work, two real-world applications of fractional order control are included: an unmanned aerial vehicle (UAV) flight control system and an

industrial hard-disk-drive (HDD) servo system. These are really exciting real-world applications that clearly show the advantages of using fractional calculus for motion controls.

This book is organized as follows. Part I contains only Chapter 1, introducing fundamentals of fractional order systems and controls followed by research motivations and book contributions. Part II is dedicated to the fractional order velocity controls, which includes Chapters 2–5. Part III focuses on the fractional order position controls, including Chapters 6–10. The feasible regions of the specifications for integer and fractional order controller designs based on the stability analysis are studied in Part IV, which includes Chapters 11 and 12. Part V explains how to design a fractional order disturbance observer, a fractional order adaptive feed-forward controller, a fractional order adaptive controller, and a fractional order periodic adaptive learning controller to compensate for the external disturbances in motion control systems, shown in Chapters 13–16, respectively. Part VI is devoted to the fractional order controls on nonlinear control systems in Chapters 17–19. Applications of fractional order controls in UAV flight control system and the HDD servo system are presented in Part VII including Chapters 20 and 21.

It is our sincere hope that this book can well serve two purposes. For motion control researchers and engineers, this book offers some new schemes that can present further improved performance not achievable before. For researchers and students interested in fractional calculus, this book is a demonstration that fractional calculus is indeed useful in real-world applications, not just a pure math game. Given the pervasive and ubiquitous nature of fractional calculus, we do believe that, as demonstrated in this book, even for simple motion control problems, there are ample opportunities to apply fractional calculus-based control methods. For more complex engineering and non-engineering systems, the opportunities and beneficial consequences of applying fractional calculus are limited only by our imagination.

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