



QUANTUM MECHANICS



Shivam Prabhakaran



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Preface

Quantum Mechanics is a theory of Mechanics, a branch of Physics that deals with the Motion of bodies and associated physical quantities such as Energy and Momentum. Quantum Mechanics has had enormous success in explaining many of the features of our world. The individual behaviour of the Microscopic Particles that make up all forms of matter can often only be satisfactorily described using Quantum Mechanics.

Quantum mechanics is important for understanding how individual atoms combine to form chemicals. It provides quantitative insight into chemical bonding processes by explicitly showing which molecules are energetically favourable to which others, and by approximately how much. This book is intended to provide a comprehensive coverage of the major aspects of quantum mechanics. The most likely audience for the book consists of students and teachers of modern physics, mechanics and engineering.

Shivam Prabhakaran

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Introduction to Quantum Physics

For centuries, man has wondered on phenomena and processes happening around him. As time passed, he was successful in applying his intuition and common sense in comprehending the stars, galaxies and their behaviour, but they fail in the microscopic world of molecules, atoms and sub-atomic particles.

Quantum theory provides us with the rules and regulations of the miniature world. These rules are phenomenally successful in accounting for the properties of atoms, molecules, and their constituents, and form the basis of understanding the fundamental properties of all matter. In fact, one may say that the greatest success story of the 20th-century physics is to confirm that this theory works, without a single exception, in spite of critical examination by some of the best minds spanning decades of time.

The conceptual foundation of quantum theory is mysterious. It led to intense debates among scientists, and confused many. Niels Bohr, one of the most prominent scientists in this domain, once remarked, “You have not studied quantum mechanics well if you aren’t confused by it.” Albert Einstein, the greatest physicist of the 20th century, never approved of this theory. Bizarre though it may seem, quantum physics has led physicists step by step to a deeper view of the reality, and has answered many fundamental questions.

Quantum physics is a branch of science that deals with discrete, indivisible units of energy called quanta as described by the Quantum Theory. There are five main ideas represented in Quantum Theory:

1. Energy is not continuous, but comes in small but discrete units.

2. The elementary particles behave both like particles and like waves.
3. The movement of these particles is inherently random.
4. It is *physically impossible* to know both the position and the momentum of a particle at the same time.
5. The atomic world is nothing like the world we live in.

While at a glance this may seem like just another strange theory, it contains many clues as to the fundamental nature of the universe and is more important than even relativity in the grand scheme of things (if any one thing at that level could be said to be more important than anything else). Furthermore, it describes the nature of the universe as being much different than the world we see. As Niels Bohr said, “Anyone who is not shocked by quantum theory has not understood it.”

Particle / Wave Duality

Particle/wave duality is perhaps the easiest way to get acquainted with quantum theory because it shows, in a few simple experiments, how different the atomic world is from our world.

First let's set up a generic situation to avoid repetition. In the centre of the experiment is a wall with two slits in it. To the right we have a detector. What exactly the detector is varies from experiment to experiment, but its purpose stays the same: detect how many of whatever we are sending through the experiment reaches each point. To the left of the wall we have the originating point of whatever it is we are going to send through the experiment. That's the experiment: send something through two slits and see what happens. For simplicity, assume that nothing bounces off of the walls in funny patterns to mess up the experiment.

First try the experiment with bullets. Place a gun at the originating point and use a sandbar as the detector. First try covering one slit and see what happens. You get more bullets near the centre of the slit and less as you get further away. When you cover the other slit, you see the same thing with respect to the other slit. Now open both slits. You get the sum of the result of opening each slit. The most bullets are found in the middle of the two slits with less being found the further you get from the centre.

Well, that was fun. Let's try it on something more interesting: water waves. Place a wave generator at the originating point and detect using a wave detector that measures the height of the waves that pass. Try it with one slit closed. You see a result just like that of the bullets. With the other slit closed the result is the same. Now try it with both slits open. Instead of getting the sum of the results of each slit being open, you see a wavy pattern; in the centre there is a wave greater than the sum of what appeared there each time only one slit was open. Next to that large wave was a wave much smaller than what appeared there during either of the two single slit runs. Then the pattern repeats; large wave, though not nearly as large as the centre one, then small wave. This makes sense; in some places the waves reinforced each other creating a larger wave, in other places they canceled out. In the centre there was the most overlap, and therefore the largest wave. In mathematical terms, instead of the resulting intensity being the sum of the squares of the heights of the waves, it is the square of the sum.

While the result was different from the bullets, there is still nothing unusual about it; everyone has seen this effect when the waves from two stones that are dropped into a lake in different places overlap. The difference between this experiment and the previous one is easily explained by saying that while the bullets each went through only one slit, the waves each went through both slits and were thus able to interfere with themselves.

Now try the experiment with electrons. Recall that electrons are negatively charged *particles* that make up the outer layers of the atom. Certainly they could only go through one slit at a time, so their pattern should look like that of the bullets, right? Let's find out. Place an electron gun at the originating point and an electron detector in the detector place. First try opening only one slit, then just the other. The results are just like those of the bullets and the waves. Now open both slits. *The result is just like the waves.*

There must be some explanation. After all, an electron couldn't go through both slits. Instead of a continuous stream of electrons, let's turn the electron gun down so that at any one time only one electron is in the experiment. Now the electrons won't be able to cause trouble since there is no one else to interfere with. The result should now look like the bullets. But it doesn't! It would seem that the electrons do go through both slits.

This is indeed a strange occurrence; we should watch them ourselves to make sure that this is indeed what is happening. So, we put a light behind the wall so that we can see a flash from the slit that the electron went through, or a flash from both slits if it went through both. Try the experiment again. As each electron passes through, there is a flash in only one of the two slits.

Obviously the light is causing problems. Perhaps if we turned down the intensity of the light, we would be able to see them without disturbing them. When we try this, we notice first that the flashes we see are the same size. Also, some electrons now get by without being detected. This is because light is not continuous but made up of particles called photons. Turning down the intensity only lowers the number of photons given out by the light source. The particles that flash in one slit or the other behave like the bullets, while those that go undetected behave like waves.

Well, we are not about to be outsmarted by an electron, so instead of lowering the intensity of the light, why don't we lower the frequency. The lower the frequency the less the electron will be disturbed, so we can finally see what is actually going on. Lower the frequency slightly and try the experiment again. We see the bullet curve. After lowering it for a while, we finally see a curve that looks somewhat like that of the waves! There is one problem, though. Lowering the frequency of light is the same as increasing its wavelength, and by the time the frequency of the light is low enough to detect the wave pattern the wavelength is longer than the distance between the slits so we can no longer see which slit the electron went through.

So have the electrons outsmarted us? Perhaps, but they have also taught us one of the most fundamental lessons in quantum physics - an observation is only valid in the context of the experiment in which it was performed. If you want to say that something behaves a certain way or even exists, you must give the context of this behaviour or existence since in another context it may behave differently or not exist at all. We can't just say that an electron is a particle, since we have already seen proof that this is not always the case. We can only say that when we observe the electron in the two slit experiment it behaves like a particle. To see how it would behave under different conditions, we must perform a different experiment.

The Copenhagen Interpretation

So sometimes a particle acts like a particle and other times it acts like a wave. So which is it? According to Niels Bohr, who worked in Copenhagen when he presented what is now known as the Copenhagen interpretation of quantum theory, the particle is what you measure it to be. When it looks like a particle, it is a particle. When it looks like a wave, it is a wave. Furthermore, it is meaningless to ascribe any properties or even existence to anything that has not been measured. Bohr is basically saying that nothing is real unless it is observed.

While there are many other interpretations of quantum physics, all based on the Copenhagen interpretation, the Copenhagen interpretation is by far the most widely used because it provides a 'generic' interpretation that does not try to say any more than can be proven. Even so, the Copenhagen interpretation does have a flaw that we will discuss later. Still, since after 70 years no one has been able to come up with an interpretation that works better than the Copenhagen interpretation, that is the one we will use. We will discuss one of the alternatives later.

The Wave Function

In 1926, just weeks after several other physicists had published equations describing quantum physics in terms of matrices, Erwin Schrodinger created quantum equations based on wave mathematics, a mathematical system that corresponds to the world we know much more than the matrices. After the initial shock, first Schrodinger himself then others proved that the equations were mathematically equivalent. Bohr then invited Schrodinger to Copenhagen where they found that Schrodinger's waves were in fact nothing like real waves. For one thing, each particle that was being described as a wave required three dimensions. Even worse, from Schrodinger's point of view, particles still jumped from one quantum state to another; even expressed in terms of waves space was still not continuous. Upon discovering this, Schrodinger remarked to Bohr that "Had I known that we were not going to get rid of this damned quantum jumping, I never would have involved myself in this business."

Unfortunately, even today people try to imagine the atomic world as being a bunch of classical waves. As Schrodinger found out, this

could not be farther from the truth. The atomic world is nothing like our world, no matter how much we try to pretend it is. In many ways, the success of Schrodinger's equations has prevented people from thinking more deeply about the true nature of the atomic world.

The Collapse of the Wave Function

So why bring up the wave function at all if it hampers full appreciation of the atomic world? For one thing, the equations are much more familiar to physicists, so Schrodinger's equations are used much more often than the others. Also, it turns out that Bohr liked the idea and used it in his Copenhagen interpretation. Remember the experiment with electrons? Each possible route that the electron could take, called a ghost, could be described by a wave function. As we shall see later, the 'damned quantum jumping' insures that there are only a finite, though large, number of possible routes. When no one is watching, the electron take every possible route and therefore interferes with itself. However, when the electron is observed, it is forced to choose one path. Bohr called this the "collapse of the wave function". The probability that a certain path will be chosen when the wave function collapses is essentially the square of the path's wave function.

Bohr reasoned that nature likes to keep its possibilities open, and therefore follows every possible path. Only when observed is nature forced to choose only one path, so only then is just one path taken.

The Uncertainty Principle

If we are going to destroy the wave pattern by observing the experiment, then we should at least be able to determine exactly where the electron goes. Newton figured that much out back in the early eighteenth century; just observe the position and momentum of the electron as it leaves the electron gun and we can determine exactly where it goes.

Well, fine. But how exactly are we to determine the position and the momentum of the electron? If we disturb the electrons just in seeing if they are there or not, how are we possibly going to determine both their position and momentum? Still, a clever enough person, say Albert Einstein, should be able to come up with something, right?

Unfortunately not. Einstein did actually spend a good deal of his life trying to do just that and failed. Furthermore, it turns out that if it were possible to determine both the position and the momentum at the same time, Quantum Physics would collapse. Because of the latter, Werner Heisenberg proposed in 1925 that it is in fact *physically impossible* to do so. As he stated it in what now is called the Heisenberg Uncertainty Principle, if you determine an object's position with uncertainty x , there must be an uncertainty in momentum p , such that $xp > h/4\pi$, where h is Planck's constant (which we will discuss shortly). In other words, you can determine *either* the position *or* the momentum of an object as accurately as you like, but the act of doing so makes your measurement of the other property that much less. Human beings may someday build a device capable of transporting objects across the galaxy, but no one will *ever* be able to measure both the momentum and the position of an object at the same time. This applies not only to electrons but also to objects such as tennis balls and toasters, though for these objects the amount of uncertainty is so small compared to their size that it can safely be ignored under most circumstances.

The EPR Experiment

"God does not play dice" was Albert Einstein's reply to the Uncertainty Principle. Thus being his belief, he spent a good deal of his life after 1925 trying to determine both the position and the momentum of a particle. In 1935, Einstein and two other physicists, Podolski and Rosen, presented what is now known as the EPR paper in which they suggested a way to do just that. The idea is this: set up an interaction such that two particles go off in opposite directions and do not interact with anything else. Wait until they are far apart, then measure the momentum of one and the position of the other. Because of conservation of momentum, you can determine the momentum of the particle not measured, so when you measure its position you know both its momentum and position. The only way quantum physics could be true is if the particles could communicate faster than the speed of light, which Einstein reasoned would be impossible because of his Theory of Relativity.

In 1982, Alain Aspect, a French physicist, carried out the EPR experiment. He found that *even if information needed to be*

communicated faster than light to prevent it, it was not possible to determine both the position and the momentum of a particle at the same time. This does not mean that it is possible to send a message faster than light, since viewing either one of the two particles gives no information about the other. It is only when both are seen that we find that quantum physics has agreed with the experiment. So does this mean relativity is wrong? No, it just means that the particles do not communicate by any means we know about. All we know is that every particle knows what every other particle it has ever interacted with is doing.

The Quantum and Planck's Constant

So what is that h that was so important in the Uncertainty Principle? Well, technically speaking, its 6.63×10^{-34} joule-seconds. It's called Planck's constant after Max Planck who, in 1900, introduced it in the equation $E=h\nu$ where E is the energy of each quantum of radiation and ν is its frequency. What this says is that energy is not continuous as everyone had assumed but only comes in certain finite sizes based on Planck's constant.

At first physicists thought that this was just a neat mathematical trick Planck used to explain experimental results that did not agree with classical physics. Then, in 1904, Einstein used this idea to explain certain properties of light—he said that light was in fact a particle with energy $E=h\nu$. After that the idea that energy isn't continuous was taken as a fact of nature—and with amazing results. There was now a reason why electrons were only found in certain energy levels around the nucleus of an atom. Ironically, Einstein gave quantum theory the push it needed to become the valid theory it is today, though he would spend the rest of his life trying to prove that it was not a true description of nature.

Also, by combining Planck's constant, the constant of gravity, and the speed of light, it is possible to create a quantum of length (about 10^{-35} metre) and a quantum of time (about 10^{-43} sec), called, respectively, Planck's length and Planck's time. While saying that energy is not continuous might not be too startling to the average person, since what we commonly think of as energy is not all that well defined anyway, it is startling to say that there are quantities of space and time that cannot be broken up into smaller pieces. Yet it is exactly this that gives nature a finite number of routes to take when an electron interferes with itself.

Although it may seem like the idea that energy is quantized is a minor part of quantum physics when compared with ghost electrons and the uncertainty principle, it really is a fundamental statement about nature that caused everything else we've talked about to be discovered. And it is always true. In the strange world of the atom, anything that can be taken for granted is a major step towards an 'atomic worldview'.

Schrodinger's Cat

There was a problem with the Copenhagen interpretation? Well, you now know enough of what quantum physics *is* to be able to discuss what it *isn't*, and by far the biggest thing it isn't is complete. Sure, the math seems to be complete, but the theory includes absolutely nothing that would tie the math to any physical reality we could imagine. Furthermore, quantum physics leaves us with a rather large open question: *what is reality?* The Copenhagen interpretation attempts to solve this problem by saying that reality is what is measured. However, the measuring device itself is then not real until *it* is measured. The problem, which is known as the measurement problem, is when does the cycle stop?

Remember that when we last left Schrodinger he was muttering about the 'damned quantum jumping.' He never did get used to quantum physics, but, unlike Einstein, he was able to come up with a very real demonstration of just how incomplete the physical view of our world given by quantum physics really is. Imagine a box in which there is a radioactive source, a Geiger counter (or anything that records the presence of radioactive particles), a bottle of cyanide, and a cat. The detector is turned on for just long enough that there is a fifty-fifty chance that the radioactive material will decay. If the material does decay, the Geiger counter detects the particle and crushes the bottle of cyanide, killing the cat. If the material does not decay, the cat lives. To us outside the box, the time of detection is when the box is open. At that point, the wave function collapses and the cat either dies or lives. However, until the box is opened, the cat is both dead and alive.

On one hand, the cat itself could be considered the detector; its presence is enough to collapse the wave function. But in that case, would the presence of a rat be enough? Or an amoeba? Where is the line drawn? On the other hand, what if you replace the cat with a human (named 'Wigner's friend' after Eugene Wigner, the physicist

who developed many derivations of the Schrodinger's cat experiment). The human is certainly able to collapse the wave function, yet to us outside the box the measurement is not taken until the box is opened. If we try to develop some sort of 'quantum relativity' where each individual has his own view of the world, then what is to prevent the world from getting "out of sync" between observers?

While there are many different interpretations that solve the problem of Schrodinger's Cat, one of which we will discuss shortly, none of them are satisfactory enough to have convinced a majority of physicists that the consequences of these interpretations are better than the half dead cat. Furthermore, while these interpretations do prevent a half dead cat, they do not solve the underlying measurement problem. Until a better interpretation surfaces, we are left with the Copenhagen interpretation and its half dead cat. We can certainly understand how Schrodinger feels when he says, "I don't like it, and I'm sorry I ever had anything to do with it."

The Infinity Problem

There is one last problem that we will discuss before moving on to the alternative interpretation. Unlike the others, this problem lies primarily in the mathematics of a certain part of quantum physics called quantum electrodynamics, or QED. This branch of quantum physics explains the electromagnetic interaction in quantum terms. The problem is, when you add the interaction particles and try to solve Schrodinger's wave equation, you get an electron with infinite mass, infinite energy, and infinite charge. There is no way to get rid of the infinities using valid mathematics, so, the theorists simply divide infinity by infinity and get whatever result the guys in the lab say the mass, energy, and charge should be. Even fudging the math, the other results of QED are so powerful that most physicists ignore the infinities and use the theory anyway. As Paul Dirac, who was one of the physicists who published quantum equations before Schrodinger, said, "Sensible mathematics involves neglecting a quantity when it turns out to be small—not neglecting it just because it is infinitely great and you do not want it!"

Many Worlds

One other interpretation, presented first by Hugh Everett III in 1957, is the many worlds or branching universe interpretation. In this theory, whenever a measurement takes place, the entire universe

divides as many times as there are possible outcomes of the measurement. All universes are identical except for the outcome of that measurement. Unlike the science fiction view of ‘parallel universes’, it is not possible for any of these worlds to interact with each other.

While this creates an unthinkable number of different worlds, it does solve the problem of Schrodinger’s cat. Instead of one cat, we now have two; one is dead, the other alive. However, it has still not solved the measurement problem. If the universe split every time there was more than one possibility, then we would not see the interference pattern in the electron experiment. So when does it split? No alternative interpretation has yet answered this question in a satisfactory way.

Classical Physics from Newton to Einstein

The Scientific Method

The scientific method has four major components:

1. The assumption of an external, objective reality that can be observed.
2. Quantitative experiments on the external objective reality in order to determine its observable properties, and the use of induction to discover its general principles.
3. Validation of the results of these measurements by widespread communication and publication so that other scientists are able to verify them independently. Although scientists throughout history have communicated and published their results, the first scientist to articulate the need for publishing the details of his experimental methods so that other scientists could repeat his measurements was English chemist Robert Boyle, who was strongly influenced by the views of Bacon.
4. Intuiting and formulating the mathematical laws that describe the external objective reality. The most universal laws are those of physics, the most fundamental science. English natural philosopher Isaac Newton was the first scientist to formulate laws that were considered to apply universally to all physical systems.

The last three of these components were all developed in the remarkably brief period from 1620 to 1687, and all by Englishmen!

Newton's Laws and Determinism

In order to understand quantum physics, we must first understand classical physics so that we can see the differences between them.

There are two fundamental assumptions in classical physics. The first fundamental assumption is that the objective world exists independently of any observations that are made on it. To use a popular analogy, a tree falling in the forest produces a sound whether or not it is heard by anyone. While it is possible that observations of the objective world can affect it, its independence guarantees that they do not necessarily affect it.

The second fundamental assumption of classical physics is that both the position and velocity of an object can be measured with no limits on their precision except for those of the measuring instruments. In other words, the objective world is a precise world with no intrinsic uncertainty in it. As we shall see later, quantum theory abandons both of these fundamental assumptions.

Isaac Newton was the first important scientist both to do fundamental experiments and to devise comprehensive mathematical theories to explain them. He invented a theory of gravity to explain the laws of German astronomer and mathematician Johannes Kepler which describe the planetary orbits, made use of the famous free-fall experiments from the leaning tower of Pisa by Italian scientist Galileo Galilei, and invented the calculus in order to give a proper mathematical framework to the laws of motion that he discovered. Newton considered himself to be a natural philosopher, but contemporary custom would accord him the title of physicist. Indeed, he, probably more than any other scientist, established physics as a separate scientific discipline because of his attempts to express his conclusions in terms of universal physical laws.

His three laws of motion can be written as follows:

1. A body moves with constant velocity unless there is a nonzero net force acting on it.
2. The rate of change of the velocity of a body is proportional to the force on the body.

3. If one body exerts a force on another body, the second body exerts an equal and opposite force on the first.

In order to use these laws, the properties of the forces acting on a body must be known. As an example of a force and its properties, Newton's law of gravitation states that the gravitational force between two bodies, such as the earth and the moon, is proportional to the mass of each body and is inversely proportional to the square of the distance between them. This description of the gravitational force, when used together with Newton's second law, explains why the planetary orbits are elliptical. Because of Newton's third law, the force acting on the earth is equal and opposite to the force acting on the moon. Both bodies are constantly changing their speeds and directions because of the gravitational force continually acting on them.

For more than 200 years, after many experiments on every accessible topic of macroscopic nature, Newton's laws came to be regarded by physicists and by much of society as the laws that were obeyed by all phenomena in the physical world. They were successful in explaining all motions, from those of the planets and stars to those of the molecules in a gas. This universal success led to the widespread belief in the principle of determinism, which says that, if the state of a system of objects (even as all-encompassing as the universe) is known precisely at any given time, such as now, the state of the system at any time in the future can in principle be predicted precisely. For complex systems, the actual mathematics might be too complicated, but that did not affect the principle. Ultimately, this principle was thought to apply to living beings as well as to inanimate objects. Such a deterministic world was thought to be completely mechanical, without room for free will, indeed without room for even any small deviation from its ultimate destiny. If there was a God in this world, his role was limited entirely to setting the whole thing into motion at the beginning.

Intrinsic to the principle of determinism was the assumption that the state of a system of objects could be precisely described at all times. This meant, for example, that the position and velocity of each object could be specified exactly, without any uncertainty. Without such exactitude, prediction of future positions and velocities would be impossible. After many, many experiments it seemed clear that

only the inevitable imprecision in measuring instruments limited the accuracy of a velocity or position measurement, and nobody doubted that accuracies could improve without limit as measurement techniques improved.

*Thermodynamics and Statistical Mechanics,
Entropy and the Direction of Time*

Thermodynamics is the physics of heat flow and of the interconversion between heat energy and other forms of energy. Statistical mechanics is the theory that describes macroscopic properties such as pressure, volume and temperature of a system in terms of the average properties of its microscopic constituents, the atoms and molecules. Thermodynamics and statistical mechanics are both concerned with predicting the same properties and describing the same processes, thermodynamics from a macroscopic point of view, and statistical mechanics from a microscopic point of view.

In 1850, the German physicist Rudolf Clausius proposed the first law of thermodynamics, which states that energy may be converted from one form to another, such as heat energy into the mechanical rotation of a turbine, but it is always conserved. Since 1905 when German-Swiss-American physicist Albert Einstein invented the special theory of relativity, we know that energy and matter can be converted into each other. Hence, the first law actually applies jointly to both matter and energy. This law is probably the most fundamental one in nature. It applies to all systems, no matter how small or large, simple or complex, whether living or inanimate. We do not think it is ever violated anywhere in the universe. No new physical theory is ever proposed without checking to see whether it upholds this law.

The second law of thermodynamics can be stated in several ways. The first statement of it, made by Rudolf Clausius in 1850, is that heat can flow spontaneously from a hot to a cold object but it cannot spontaneously pass from a cold to a hot object. The second statement of the second law was made later by Scottish physicist William Thomson Kelvin and German physicist Max Planck: Heat energy cannot be completely transformed into mechanical energy, but mechanical energy can be completely transformed into heat energy. The third statement of the second law depends on a new concept, that of entropy.

Entropy is related to the amount of disorder and order in the system. Decreasing entropy is equivalent to decreasing disorder or disorganization (increasing order or organization) of an object or system; while increasing entropy is equivalent to increasing disorder or disorganization.

It turns out that the second law of thermodynamics can be stated in the following way: Natural processes of an isolated macroscopic system normally proceed in the direction of maximum probability (maximum disorder), which is the direction of maximum number of distinguishable arrangements of the system. (It is highly improbable, although not totally impossible, for them to proceed in the opposite direction.) The forward direction of time is the direction in which entropy increases. Thus, the second law of thermodynamics can be restated in terms of entropy: Natural processes of an isolated macroscopic system always proceed in the direction of increasing entropy (disorder).

The direction of time can also be inferred from the first two statements of the second law of thermodynamics: (1) The unidirectional flow of heat from hot to cold bodies, and (2) the possibility of total conversion of mechanical energy to heat energy, but not the reverse.

A mistake made by some people is to think that the second law applies to individual objects or systems, such as automobiles, plants, or human bodies, even if they are not isolated from the rest of the universe, and that this is the reason that such objects decay and disintegrate with time. This is a fallacy, however, because the second law does not prevent the entropy of an individual object from continuously decreasing with time and thus becoming more ordered and organized as long as it receives energy from something else in the universe whose entropy continues to increase. In our solar system, it is primarily the sun's entropy that continually increases as its fuel is burned and it becomes more disordered.

An extremely important property of Newton's laws is that they are time reversal invariant. What this obscure-sounding term means is that, if the direction of time is reversed, the directions of motion of all particles are also reversed, and this reversed motion is completely allowed by Newton's laws. In other words, the motion in reversed time is just as valid as the motion in forward time, and nature

herself does not distinguish between the two. A simple example of this is the time-reversed motion of a thrown baseball, which follows a parabolic trajectory in either the forward or the reversed direction. Without seeing the act of throwing, and without air resistance, we would not be able to distinguish the forward parabola from the reversed parabola. Another way to state it is that a movie of a thrown baseball seems just as valid to us if it is run in the reverse direction as in the forward direction. Time reversal invariance is also apparent in the seemingly random motion of the molecules in a gas. If we could see their motion in a movie and then reverse it, we could not distinguish between the forward motion and the reversed motion.

However, if we consider the motion of an object containing many ordered particles (for example, with a recognizable size, shape, position, velocity, and orientation), we encounter a different phenomenon. It is easy to tell the difference between the reversed and forward motions of a person, a horse, a growing plant, a cup falling from a table and breaking, and most other examples from everyday life. Another example is the free expansion of a gas that initially is confined to one side of a box by a membrane. If the membrane is broken, the gas immediately expands into the other side (initially assumed to be evacuated), and we can easily tell the time reversed motion from the forward motion. In all of these cases, the motion at the individual molecule level is time reversal invariant, but it is clear that the gross motion of the macroscopic object is not.

Our question now is, "Why does nature seem to be time reversal invariant at the individual, or few, particle level, but apparently not at the level of many particles contained in an ordered system such as any common macroscopic object?" In classical physics, irreversibility is always due to the second law of thermodynamics, which determines the forward direction of time. The forward direction is apparent after the cup has fallen and broken because the broken cup is more disordered (has higher entropy) than the unbroken cup. However, even before the cup breaks, a detailed calculation would show that the entropy of the combined system of cup, gravitational force, and earth increases as the cup falls. The entropy of the system of moving horse or person, gravitational force, earth, and surroundings increases with time because the motion dissipates energy and increases the disorder in the body, earth, and surroundings.

Electromagnetism

French physicist Charles Augustin de Coulomb discovered the force law obeyed by stationary, electrically charged objects between 1785 and 1791. In 1820, Danish physicist Hans Christian Oersted discovered that an electric current produces a magnetic field, and showed that a magnetic field exerted a force on a current-carrying wire. From 1820 to 1827, French physicist Andre Ampere extended these discoveries and developed the mathematical relationship describing the strength of the magnetic field as a function of current. In 1831, English chemist and physicist Michael Faraday discovered that a changing magnetic field, which he explained in terms of changing magnetic lines of force, produces an electric current in a wire. This was a giant step forward because it was the forerunner of the concept of force fields, which are used to explain all forces in nature today.

These disparate phenomena and theories were all pulled together into one elegant theory by Scottish physicist James Clark Maxwell in 1873. Maxwell's four equations describing the electromagnetic field are recognized as one of the great achievements of 19th century physics. Maxwell was able to calculate the speed of propagation of the electromagnetic field from his equations, and found it to be approximately equal to the speed of light. He then proposed that light is an electromagnetic phenomenon. Because electromagnetic fields can oscillate at any frequency, he concluded that visible light occupied only a very small portion of the frequency spectrum of electromagnetic radiation. The entire spectrum includes radio waves of low-frequency, high-frequency, very-high frequency, ultra-high frequency, and microwaves. At still higher frequencies are infrared radiation, visible light, ultraviolet radiation, x-rays, and gamma rays. All of these are fundamentally the same kind of waves, the only difference between them being the frequency of the radiation.

Now we ask, what is the electromagnetic field, anyway? Is it a physical object? To answer that question, we must understand what we mean by the term physical object. One definition is that it is anything that carries force, energy, and momentum. By this definition the electromagnetic field is a physical object because it carries force, energy, and momentum. However, this merely defines the electromagnetic field in terms of other things that require their own

definitions. Force, energy, and momentum can only be defined in terms of the operations necessary to measure them and these operations require physical objects on which to make the measurements. Thus, all physical objects are defined in terms of other physical objects, so the definition is circular. This is another indication that the concept of objective reality is nothing but a concept.

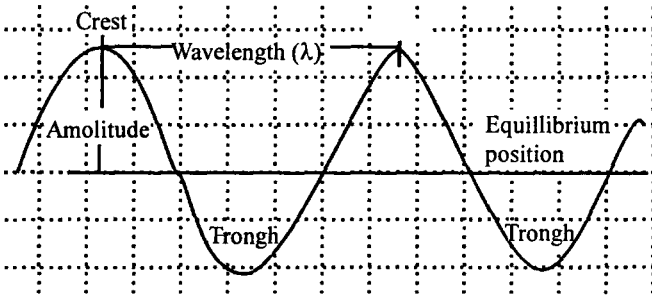


Fig. 1 Waves

These parameters are related by the following equation: $v = \lambda f$

The electromagnetic spectrum contains electromagnetic waves of all frequencies and wavelengths:

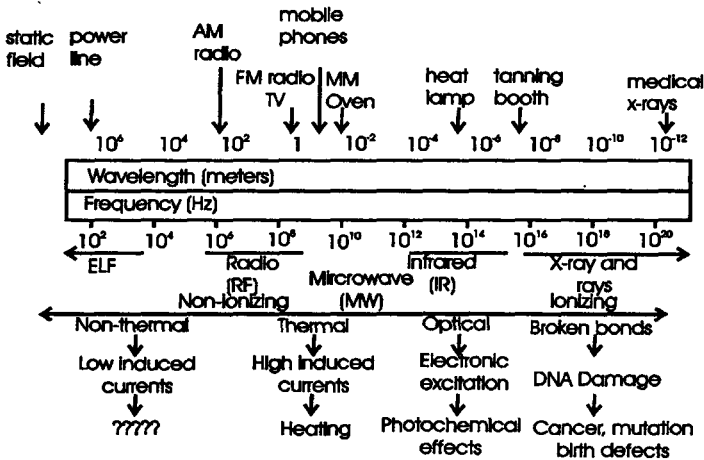


Fig.2 Electromagnetic waves of all frequencies and wavelengths

Waves

In the 1800s, it was known that light had a wave-like nature, and classical physics assumed that it was indeed a wave. Waves are traveling oscillations. Examples are water waves, which are traveling surface

oscillations of water; and waves on a tightly stretched rope, which are traveling oscillations of the rope. Waves are characterized by three parameters: wavelength (λ), oscillation frequency (f), and velocity (v).

It was not known what the oscillating medium was in the case of light, but it was given the name 'ether.' Maxwell had assumed that the ether provided an absolute reference frame with respect to which the velocity of any object or wave could be measured. In 1881, German-American physicist Albert Michelson and American physicist Edward Morley performed ground-breaking experiments on the velocity of light. They found that the velocity of light on the earth always had the same constant value regardless of the direction of motion of the earth about the sun. This violated the concept, which was prevalent at the time, that the measured velocity of any object, be it particle or wave, depends on the observer's velocity relative to the velocity of the other object. This concept is demonstrated in everyday life when our observation of another car's velocity depends on the velocity of our own car. Thus, the measured velocity of light relative to the ether was expected to depend on the direction of motion of the earth relative to the velocity of the ether. But, the constancy of the velocity of light meant that the concept of the ether had to be abandoned because the ether velocity could not be expected to change with the observer's velocity in just such a way that the velocity of light always had the same value. Thus, in the case of light waves, physicists concluded that there is no material medium that oscillates.

Relativity

Implicit in the preceding discussion of classical physics was the assumption that space and time were the contexts in which all physical phenomena took place. They were absolute in the sense that no physical phenomena or observations could affect them, therefore they were always fixed and constant.

In 1905, the German-Swiss-American physicist Albert Einstein revolutionized these ideas of time and space by publishing his theory of special relativity. In this theory, he abandoned the concept of the ether, and with that the concept of the absolute motion of an object, realizing that only relative motion between objects could be measured. Using only the assumption of the constancy of the velocity of light

in free space, he showed that neither length nor time is absolute. This means that both length and time measurements depend on the relative velocities of the observer and the observed.

An observer standing on the ground measuring the length of an airplane that is flying by will obtain a minutely smaller value than that obtained by an observer in the airplane. An observer on earth comparing a clock on a spaceship with his clock on earth will see that the spaceship clock moves slower than the earth clock.

For an object having a mass, the special theory produced the famous relationship between the total energy (E) of the object, which includes its kinetic energy, and its mass (m):

$$E = mc^2$$

where c is the velocity of light in a vacuum. Einstein's special theory has been confirmed by thousands of experiments, both direct and indirect.

In Einstein's special theory of relativity, even though space and time were no longer separately absolute, they were still Euclidean. This meant that two straight lines in space-time which were parallel at one point always remained parallel no matter what the gravitational forces were.

In 1915, Einstein completed his greatest work, the general theory of relativity. Whereas the special theory deals with objects in uniform relative motion, i.e., moving with constant speed along straight lines relative to each other, the general theory deals with objects that are accelerating with respect to each other, i.e., moving with changing speeds or on curved trajectories. Examples of accelerating objects are an airplane taking off or landing, a car increasing or decreasing its speed, an elevator starting up or coming to a stop, a car going around a curve at constant speed, and the earth revolving around the sun or the moon revolving around the earth at constant speed.

A particularly important example of acceleration is that of an object free-falling in the earth's gravity. A free-falling object is one that is acted upon only by the gravitational force, without air friction or other forces. All free-falling objects at the same spot in the earth's gravitational field fall with the same acceleration, independent of the mass or material of the object. A free-falling object, such as an

astronaut in a spaceship, does not experience a gravitational force (i.e., he/she experiences weightlessness), hence we can say that the acceleration of free-fall cancels out the gravitational force. Another way to state this fact is that a gravitational force is equivalent to an acceleration in the same direction. This is Einstein's famed equivalence postulate, which he used in discovering general relativity.

The equivalence postulate applies to all objects, even light beams. Consequently, the path of a light beam is affected by a gravitational field just like the trajectory of a baseball. However, because of the very high speed of the photons in a light beam (3×10^8 metres/second, or 186,000 miles/second), their trajectories are bent by only very tiny amounts in the gravitational fields of ordinary objects like the sun.

Because all types of objects are affected in exactly the same way by gravity, an equivalent way of looking at the problem is to replace all gravitational forces by curved trajectories. The curved trajectories are then equivalent to curving space itself! This is the second key concept that Einstein used in the general theory of relativity. The result is that the general theory replaces the concept of gravity with the curvature of space. The curvature of a light beam around an individual star or galaxy is very small and difficult to measure. Even the whole universe curves the trajectory of a light beam only a little.

Clear evidence that the force of gravity is nothing but a concept is given by the fact that it can be replaced by another concept, the concept of the curvature of space. Less clear is that the body sensations that we normally associate with the force of gravity are also nothing but concepts.

Speaking of the universe as a whole, what are the effects of curved space? The principal effect is that light beams no longer travel in straight lines. Hence, if two light beams start out parallel, they will eventually either converge or diverge. If they diverge, we say that space has negative curvature, and if they converge, we say that it has positive curvature. Zero curvature corresponds to parallel light beams always remaining parallel. This implies a Euclidean, or flat, space.

The electromagnetic field is nothing but a concept, we can now say that space is also nothing but a concept! It is a concept that allows us to conceptualize the separation of objects (which are nothing but concepts) and it allows us to predict the trajectories of light beams.

The curvature of the universe as a whole depends on the average mass density and on the expansion rate of the universe. The fact that the universe is expanding was discovered by American astronomer Edwin Hubble in 1929, 14 years after Einstein published his general theory of relativity.

Whether the space of our universe has positive or negative curvature is a matter for experimental determination. In practice, it is too difficult to do this by measuring the curvature of light beam trajectories, but the curvature can be calculated if the average mass density and the expansion velocity are known. The average mass density cannot easily be measured directly because we are unable to see matter that is not emitting light, so the average mass density in a galaxy, for example, must be calculated from the trajectories of the motion of visible stars in the galaxy. Such measurements indicate that there is a large amount of matter in the universe that does not shine with its own or reflected light. This is called dark matter.

Until 1998, it was thought that the universe was expanding at a constant rate, but in 1998 it was discovered that it is actually expanding at an accelerating rate rather than a constant one. This acceleration cannot be explained if the universe contains only ordinary and dark matter because these produce a gravitational force which is attractive, whereas an accelerating expansion requires a repulsive force. This repulsive force represents a 'dark energy' density in addition to the energy densities of ordinary and dark matter. Both dark matter and dark energy are presently being intensively investigated both theoretically and experimentally because they could be the result of new physical laws operating.

There are powerful theoretical reasons for believing that the curvature of our space is neither positive nor negative but is exactly zero. Zero curvature requires a certain value of the average mass density including both visible and dark matter. A larger value implies a positive curvature, and a smaller value implies a negative curvature. The density of visible matter by itself is not high enough to produce a zero or positive curvature.