Rolf Kindmann, Matthias Kraus
Steel Structures
Design using FEM

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Prof. Dr.-Ing. Rolf Kindmann
Ruhr-Universität Bochum
Lehrstuhl für Stahl-, Holz- und Leichtbau
Universitätsstraße 150
44801 Bochum

Dr.-Ing. Matthias Kraus
Ingenieursozietät SKP
Prinz-Friedrich-Karl-Str. 36
44135 Dortmund

Language Polishing by Paul Beverley, London

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Preface

Steel structures are usually beam or plate structures consisting of thin-walled cross sections. For their design, deformations, internal forces and moments as well as stresses are needed, and the stability of the structures is of great importance. Generally, the finite element method (FEM) is used for structural analysis and as a basis for the verification of sufficient load-bearing capacity.

This book presents the relevant procedures and methods needed for calculations, computations and verifications according to the current state of the art in Germany and the rest of Europe. In doing so the following topics are treated in detail:

- determination of cross-section properties, stresses and plastic cross section bearing capacity
- finite element method for linear and nonlinear calculations of beam structures
- solution of eigenvalue problems (stability) for flexural, lateral torsional, torsional and plate buckling
- verification of sufficient load-bearing capacity
- finite element method for open and hollow cross sections

The basis of the calculations and verifications are the German standard DIN 18800 and the German version of Eurocode 3. They are widely comparable, however, the final version of Eurocode 3 has just been published and the corresponding national annexes have to be considered.

This book has evolved from the extensive experience of the authors in designing and teaching steel structures. It is used as lecture notes for the lecture "Computer-oriented Design of Steel Structures" on the Masters' programme "Computational Engineering" at the Ruhr-University Bochum, Germany. Large parts of the contents were taken from German books – see [25], [31] and [42] – and therefore, the references at the end of the book contain many publications in the German language. Further information can be found at www.kindmann.de, www.rub.de/stahlbau and www.skp-ing.de.

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Rolf Kindmann
Matthias Kraus

Content

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1 Introduction

1.1 Verification Methods

For civil engineering structures the ultimate limit state (structural safety) and serviceability limit state have to be verified, see for example DIN 18800 Part 1. Since components for steel constructions are usually rather slender and thin-walled, structural safety verifications for constructions susceptible to losing stability regarding flexural, lateral torsional and plate buckling are of major significance and therefore constitute a main focal subject in static calculations. In this context, the determination of internal forces and moments, deformations and critical loads is a central task. Its solution is treated in this book using the finite element method (FEM).

The calculations and verifications have to meet the legal requirements as well as the state of the art. For steel structures the basic standard DIN 18800 and corresponding engineering standards, or Eurocode 3, have to be taken into consideration. Table 1.1 contains a compilation of the *verification methods* according to DIN 18800 and the verifications as they are generally applied. Eurocode 3 contains similar regulations.

Table 1.1 Verification procedures according to DIN 18800 and common verifications

Verification procedure	Calculation of **stresses S_d**	Calculation of **resistances R_d**	**Verifications**
Elastic-Elastic	Elastic theory ⇒ stresses σ and τ	Elastic theory ⇒ design value of yield stress $f_{y,d}$	Verification of stresses: $\sigma \le \sigma_{R,d} = f_{y,d}$ $\tau \le \tau_{R,d} = f_{y,d} / \sqrt{3}$ $\sigma_v \le \sigma_{R,d} = f_{y,d}$
Elastic-Plastic	Elastic theory ⇒ internal forces and moments N, M_y, etc.	Plastic theory ⇒ utilisation of the plastic bearing capacity of the cross sections	e.g. $M_y \le M_{pl,y,d}$ or using interaction conditions or the partial internal forces method
Plastic-Plastic	Plastic theory ⇒ internal forces and moments according to the plastic hinge or plastic zone theory	Plastic theory ⇒ utilisation of the plastic bearing capacity of the cross sections and the static system	According to the **plastic hinge theory** or according to the **plastic zone theory** (with computer programs)

The use of a *verification method* implies that the individual cross section parts (webs and flanges) can carry the compression stresses, so that no buckling occurs and a sufficient rotation capacity is provided. Assistance for the checking of the b/t relations can be found in profile tables; see for example [29]. If only longitudinal axial stresses and shear stresses occur, it is $\sigma_v = \sqrt{\sigma^2 + 3\tau^2}$. The verification of the *equivalent stress* (*verification method Elastic-Elastic*) is only required for $\sigma/\sigma_{R,d}$ **and** $\tau/\tau_{R,d}$ >

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0.5. Perfectly plastic internal forces and moments for rolled sections can be found in profile tables [29], interaction conditions and verifications using the partial internal forces method in [29] and [25].

The subscript "d" for S_d and R_d in Table 1.1 indicates that the **stresses must be determined using the design parameters** of the loads and that the design resistance has to be applied; see Section 1.7. Section 1.4 “Linear and Nonlinear Calculations” includes specifications concerning the calculation of stress and resistance.

1.2 Methods to Determine the Internal Forces and Moments

As it is generally known, internal forces and moments in statically determinate systems may be calculated with the help of **equilibrium conditions** and intersection methods. This is not possible with statically **in**determinate systems and thus another solution procedure is used, such as the **force method,** which is the classical method of structural analysis. It is appropriate for hand calculation and very straightforward since it is easy to understand in engineering terms. However, the disadvantage is that for differing structural systems many approaches must be developed and, moreover, it is completely inappropriate for many tasks.

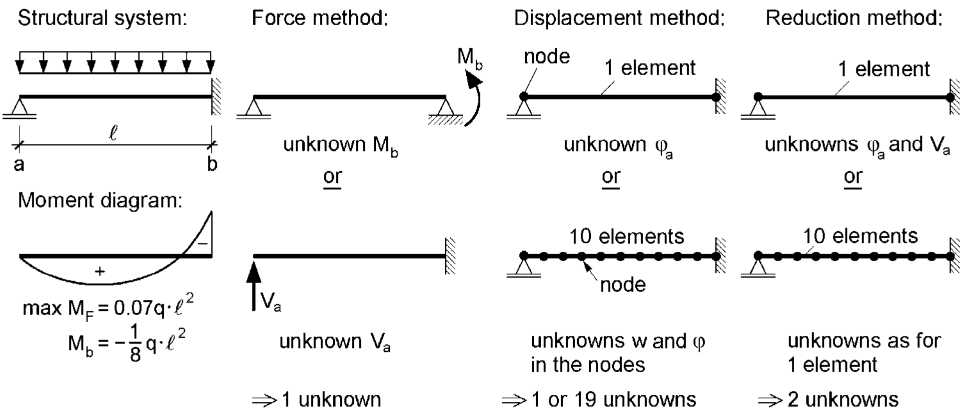


Figure 1.1 Unknown values of the force, displacement and reduction method for a selected example

Figure 1.1 exemplifies a **singlely** indeterminate girder. Hence, when using the force method, **one** unknown force value must be defined. After this, the moment distribution can be determined using the equilibrium conditions. The basis of the method is always the choice of a statically determinate structure (primary structure). Since there are several possibilities for doing so, the two systems in Figure 1.1 are selected examples.

Generally, there are three methods for determining the internal forces and moments:

- *Force method*
- *Displacement method* $\rightarrow$ FEM
- *Reduction method* $\rightarrow$ FEM

Moreover, there are numerous variations within these three methods, which cannot be discussed in detail here. Whereas when using the *force method*, the **forces** are the unknown variables of the emerging equation system, when using the displacement method, the unknown variables are the **displacements**, i.e. the displacements and rotations. If the structural system is divided into finite elements (e.g. beam elements or segments), the displacement method is extremely appropriate for a generalised formulation and so is applicable in many different situations. The ideas involved are not simple in engineer terms but are very mathematical because large amounts of data must be handled with sizable equation systems solved. The actual amount of data and the size of the equation system will, of course, depend on the system under consideration, but it will certainly be more than would be needed for the *force method*.

Figure 1.1 shows the application of the *displacement method*. Using this method, the unknown values are the deformations at the nodes, i.e. for the examined beam the displacement w and the rotation φ. Thus, there are two unknowns per node, so depending on the geometric boundary conditions, there will be between one and 19 unknowns in each example. Using the FE model with 10 elements, a relatively large number of unknowns (19) occur, but there is no need of further hand calculation, which is an advantage. For procedural reasons, all state variables (bending moments, shear forces, displacements, rotations) at the nodes, i.e. virtually in the entire system, are determined.

Due to the numeric complexity, the widespread use of the FEM with the *displacement method* is closely connected to the rapid development of high-capacity computers. Until about 1985, it was important to model structures using finite elements in such a way that the limited memory capacity was sufficient and that computing times did not escalate. Nowadays, these considerations are only important for exceptional structures and calculations. Then again, it is often seen that in static calculations exaggeratedly fine FE-modelling or the use of inappropriate finite elements create reams of paper. As shown in Figure 1.1, it can be very reasonable to calculate a single-span beam using an FEM program, since all values for the necessary verifications are directly obtained by the program and the corresponding pages for the static calculation can be printed out with minimal effort.

The third method mentioned above is the *reduction method*, which is suitable for continuous beams including instance sharp bends. The unknowns of the resulting equation system are the unknown internal forces and displacements **at the beginning** of the beam structure (see Figure 1.1), so that for beams, a maximum of seven unknowns results. Accordingly, the requirements for disc space and calculating time are low, which was, as already mentioned above, of vital importance until about 1985. The *reduction method* was often used to design plate-girder bridges, since even for multi-span girders only two unknowns arise (main beam, transfer of vertical loads). Computer programs using the *reduction method* are rare these days. However, the procedure can definitely be found in current FEM programs for beams and frameworks, though here it is first calculated with a relatively rough division into finite elements according to the *displacement method.* Subsequently, the individual beams are generally divided into five to ten elements in order to be analysed more closely using the *reduction method.* Further details on the *reduction method* can be found in [31].

1.3 Element Types and Fields of Application

For FEM calculations structures are idealised using structural systems (beams, frameworks, plates, etc.) and are then appropriately divided into finite elements – see Figure 1.3. A distinction is drawn between:

- *line elements* (one-dimensional, straight or curvilinear)
- *area elements* (two-dimensional, plane or circumflex)
- *volume elements* (three-dimensional, block-shaped or with curved surfaces)

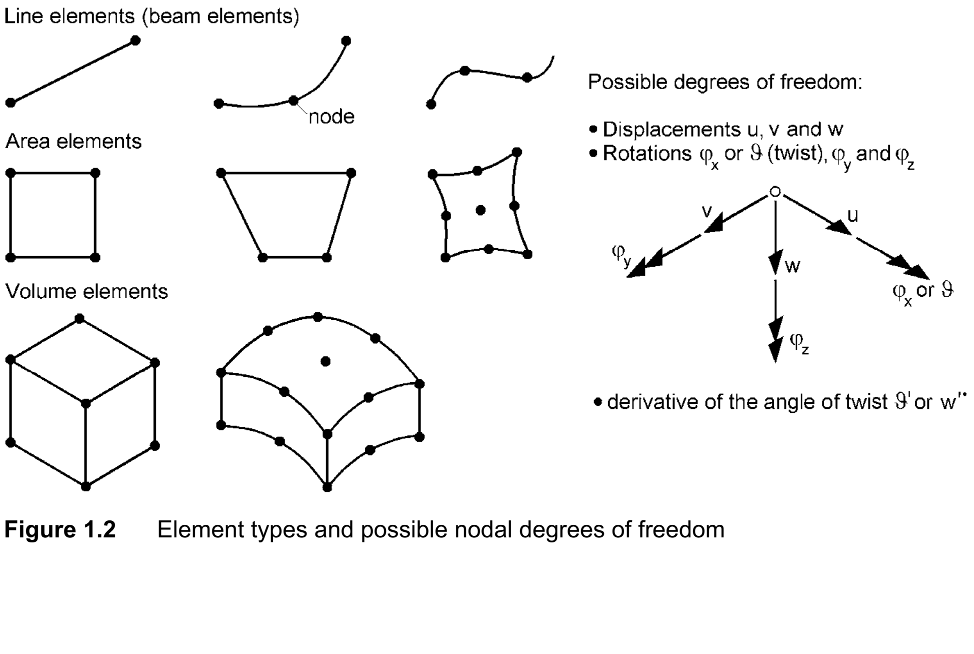


Figure 1.2 Element types and possible nodal degrees of freedom

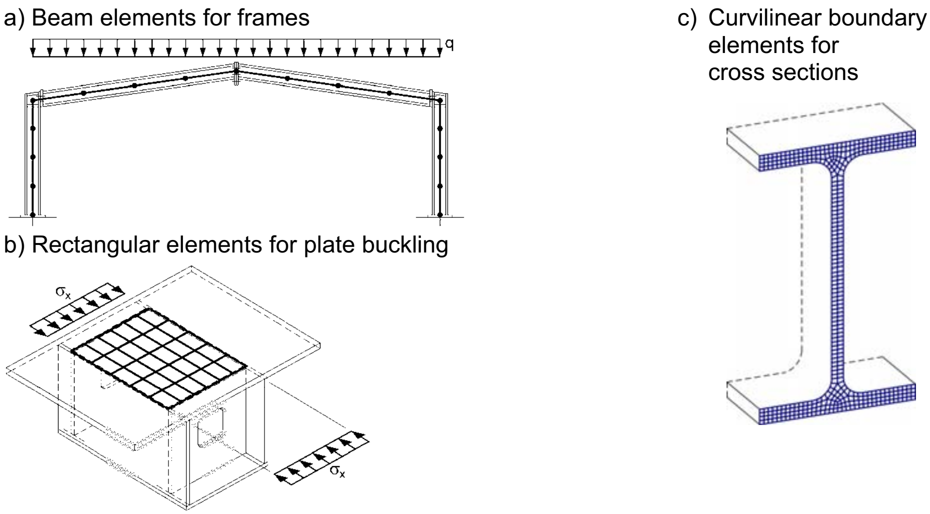


Figure 1.3 Examples for the discretisation of different problems of steel structures using finite elements

In Figure 1.2, corresponding elements are exemplified. If beams and frameworks are to be analysed, it may in some cases be useful to examine the cross section with the help of the FEM. Depending on the task, the following elements are used:

- *line elements* (one-dimensional, straight or curvilinear) or
- *area elements* (rectangular or triangular, straight or curvilinear boundaries)

For the calculation of steel structures almost exclusively beam elements are used (see Figure 1.3a). These are often part of the following structural systems:

- single-span and continuous beams
- columns and plane frames
- plane and three-dimensional trusses
- three-dimensional frameworks
- girder grids

The quoted static systems mainly appear in **structural, industrial and plant engineering**. Due to different stresses, beam elements with up to seven deformation variables in each node (degrees of freedom) are required. The number of required deformations per node is discussed in more detail in the Chapters 3 and 5.

Finite **beam elements** are also commonly used for the calculation of **bridges**. Area elements (plates, shells) are rarely used, whether for plate, beam-framework, bow or cable-stayed bridges. An essential reason for this is that the current standards and codes are almost exclusively designed to suit the needs of calculating beam structures. Moreover, apart from a few exceptions, the accuracy of these calculations is quite sufficient.

An interesting field of application for finite **area elements** in steel structures is plate buckling. For example, Figure 1.3b shows the upper flange of a beam which has been divided into finite elements in order to perform an analysis of plate buckling. This topic is discussed in Chapter 10, where a rectangular plate element for the determination of eigenvalues and modal shapes is derived. Apart from that, area elements are of course used for specific scientific research and development. Since, as has been mentioned, area elements are not often used, and **volume elements** even less so, for steel structures, the following can be stated:

- **Steel structures** are almost exclusively calculated by using **beam elements**.
- A range of beam elements are needed to appropriately calculate all occurring structures and loads.

Finite elements for the analysis of **cross sections** are covered in Chapter 11. As an example, Figure 1.3c shows the finite element modelling of a rolled I-section using area elements with curvilinear boundaries.

1.4 Linear and Nonlinear Calculations

Theoretically and numerically, linear calculations (first order theory) constitute the starting point. The following assumptions are the basis:

- The material provides a linear elastic behaviour in the whole structure, which means that *Hooke's law* is valid without restrictions of any kind.
- The influence of the deformations of the structure is so small that it may be neglected and the equilibrium conditions may be formulated for the **un**deformed structure.
- Structural and geometric imperfections, i.e. residual stresses and initial deformations, may be neglected.

Nonlinear calculations usually require a higher effort than linear calculations. Concerning the nonlinearity, we need to distinguish between **physical** and **geometric nonlinearities**. Regarding physical nonlinearity, the assumption of a linear elastic material behaviour is renounced and the plastifications in parts of the construction are considered in order to obtain more economic structures, i.e. structures of less weight. As far as the plastification is only considered regarding the bearing capacity of the cross sections, this approach is to be assigned to the **verification method Elastic-Plastic** in Table 1.1. Internal forces and moments are determined according to the elastic theory (elastic calculation of the system) and only load cases are permitted where a maximum of **one** plastic hinge occurs. In comparison to that, the plastic bearing capacity of the cross sections and the system are utilised with the **verification method Plastic-Plastic**, i.e. the spread of plastic zones or the development of several plastic hinges is permitted.

While the physical nonlinearity is mainly considered for economic reasons, the **geometrical nonlinearities** for structures susceptible to losing stability are indispensable for safety reasons. In comparison to linear calculations, relatively large deformations lead to higher internal forces and moments. For that reason, verifications against flexural, lateral torsional or plate buckling have to be executed.

In conjunction with geometric nonlinear calculations, it should be mentioned that the verifications according to the valid standards and codes, as for instance DIN 18800 Part 2, rely on a linearisation according to second order theory. This approximation is therefore the basis for the determination of deformations, internal forces and moments as well as critical loads (eigenvalues) in conformity with the codes. As a general rule, the accuracy of calculations according to second order theory is sufficient in terms of applications in engineering practice since deformations for steel structures are usually relatively small. In exceptional cases, it may be necessary to perform precise geometric nonlinear calculations. This is always the case when large or even very large deformations occur.

Summing up, the following can be stated:

- The *verification method Elastic-Elastic* is still most frequently used; see Table 1.1. For the calculation of the static system a linear elastic material behaviour is assumed with which the internal forces and moments as well as the corresponding stresses are determined. Using these stresses, the verification can be executed.
- Recently, the *verification method Elastic-Plastic* has been used more often. With this procedure, the bearing capacity can be increased until reaching the first plastic hinge.
- For structures susceptible to losing stability the geometric nonlinear problem is linearised and internal forces and moments are determined according to second order theory. This linearisation is also used for the determination of critical loads (eigenvalues).

1.5 Designations and Assumptions

In this section, descriptions and assumptions are compiled which are needed for **beam and frame structures**. Some also apply for plates and the FE analysis of cross sections. In the Chapters 10 and 11, other terms and assumptions are added relating to these topics. The basis for the designations is found in DIN 1080 and DIN 18800.

Abbreviations

ODE	ordinary differential equation
COS	coordinate system
LCC	load case combination
SMI	self moment of inertia
PIF-method	partial internal forces method
tot	total
ult	ultimate
cr	critical

Variables in the global X-Y-Z coordinate system

Beam structures are divided into beam elements, which are connected to each other at the nodes. As shown in Figure 1.2, nodes can also be arranged on the inside of an element (internal nodes). Nodes are defined in the global X-Y-Z coordinate system (COS) by using the coordinates X_k, Y_k and Z_k as shown in Figure 1.4. Moreover, all global deformations and loads at the nodes relate to this COS. For reasons of clarity, the subscript k has been neglected for these values in Figure 1.4.

The deformations in the **global COS** are marked by an **overbar** (horizontal line above the variable). This designation will also be used for vectors and matrices if they apply to the global COS.

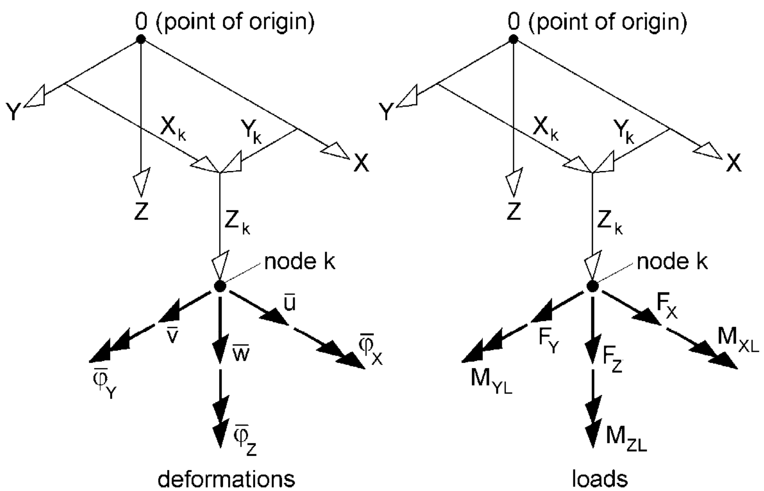
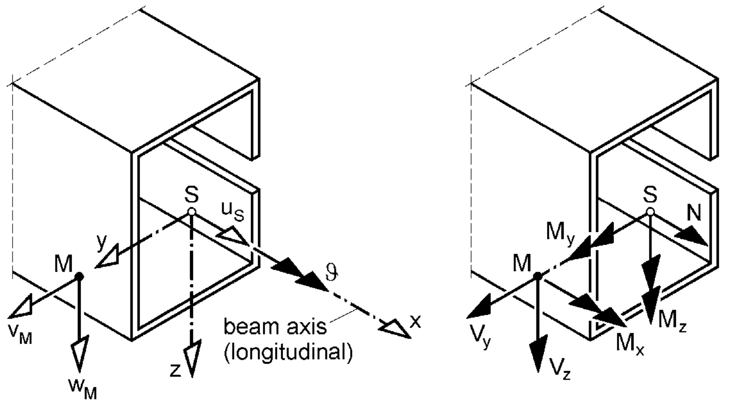


Figure 1.4 Definition of deformations and loads in the global X-Y-Z coordinate system

Variables in the local x-y-z coordinate system

Coordinates, ordinates and reference points

x	longitudinal direction of the local COS
y, z	principal axes in the cross section plane (local COS)
ω	standardised warping ordinate
S	centre of gravity
M	shear centre



beam axis x, principal axes y and z, centre of gravity S, shear centre M

Figure 1.5 Beam in the local coordinate system with displacements, internal forces and moments

Beam elements apply to a local x-y-z COS and, as longitudinal beam axis, the x-axis is defined through the centre of gravity S. The axes y and z are the principal axes of the cross section. According to Figure 1.5, some of the displacements and internal forces and moments apply to the centre of gravity S, others to the shear centre M ($y = y_M$, $z = z_M$). For warping torsion a standardised warping ordinate ω is used.

Deformation variables

u, v, w	displacements in x, y and z-direction (local COS)
$\varphi_x = \vartheta$	rotation about the x-axis (twist)
$\varphi_y \cong -w'$	rotation about the y-axis
$\varphi_z \cong v'$	rotation about the z-axis
$\psi \cong \vartheta'$	derivative of the angle of twist

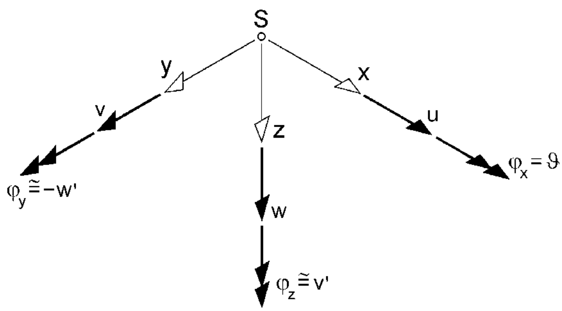


Figure 1.6 Definition of positive deformations in the local COS

Loads

q_x, q_y, q_z	distributed loads
m_x	distributed torsional moment
$M_{\omega L}$	single load warping moment

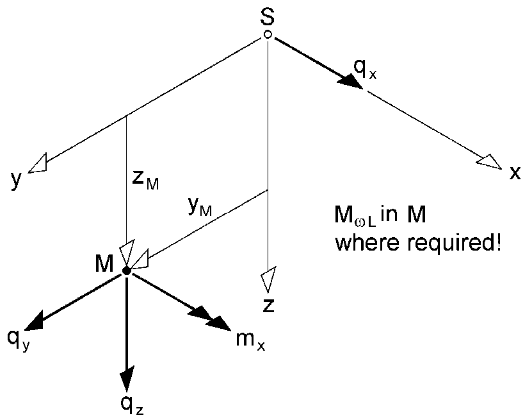


Figure 1.7 Positive directions and application points of local loads

Internal forces and moments

N	longitudinal, axial force
V_y, V_z	shear forces
M_y, M_z	bending moments
M_x	torsional moment
M_{xp}, M_{xs}	primary and secondary torsional moment
M_ω	warping bimoment
M_{rr}	see Table 5.1 (page 172)
Subscript el:	Limit internal forces and moments according to the **el**astic theory
Subscript pl:	Limit internal forces according to the **pl**astic theory
Subscript d:	**d**esign value

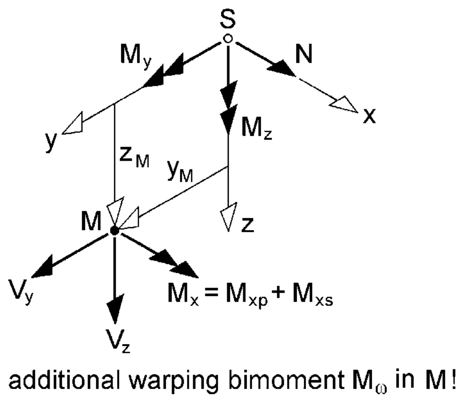


Figure 1.8 Internal forces and moments at the positive intersection of a beam

If the common definition of positive internal forces and moments (*internal force definition I*) is used, the forces at the **negative beam intersection** act in directions opposite to the ones specified in Figure 1.8. With the **sign definition II**, the directions of actions at **both beam intersections** are in compliance with the ones in Figure 1.8. In Figure 1.9, both definitions are shown for uniaxial bending with axial force.

According to custom, further subscripts are used to distinguish beam elements and nodes.

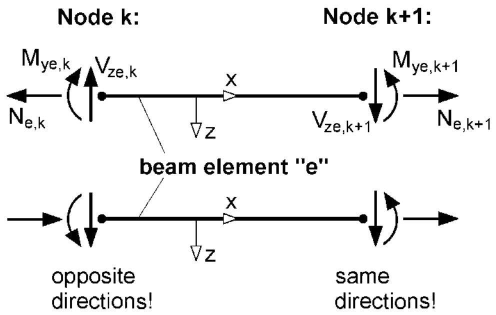


Figure 1.9 Internal forces and moments of the beam element "e" for uniaxial bending with axial force and sign definitions I and II

Stresses

σ_x, σ_y, σ_z	normal stresses
τ_{xy}, τ_{xz}, τ_{yz}	shear stresses
σ_v	equivalent stress

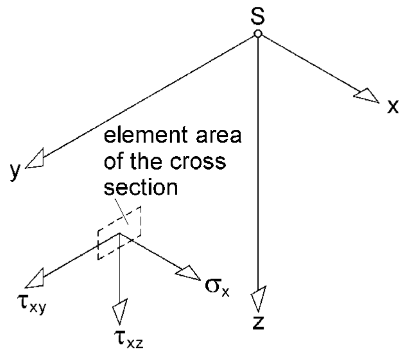


Figure 1.10 Stresses at the positive intersection of a beam

Cross section properties

A	area
I_y, I_z	principal moments of inertia
I_ω	warping constant
I_T	torsion constant (*St Venant*)
W_y, W_z	section modulus
S_y, S_z	static moments
i_M, r_y, r_z, r_ω	values for second order theory and stability; see Table 5.1
$i_p = \sqrt{\frac{I_y + I_z}{A}}$	polar radius of gyration (inertia)

Further symbols and assumptions

Material properties (isotropic material)

E	modulus of elasticity, *Young's* modulus
G	shear modulus
ν	transverse contraction, *Poisson's* ratio
f_y	yield strength, yield stress
f_u	ultimate tensile strength
ε_u	ultimate strain

Partial safety factors

γ_M	factor for resistances (**m**aterial)
γ_F	factor for loads (**f**orce)

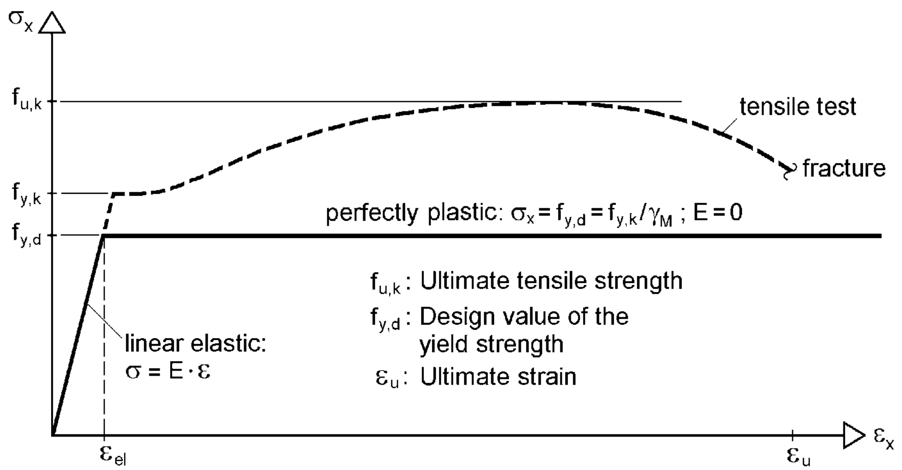


Figure 1.11 Assumptions for material behaviour

Matrices and vectors

$\underline{s}$	vector of internal forces and moments
$\underline{K}$	stiffness matrix
$\underline{G}$	geometric stiffness matrix
$\underline{v}$	vector of deformations
$\underline{p}$	load vector
subscript e:	element

An overbar above the matrices and vectors indicates that they refer to the global coordinate system (X, Y, Z).

As long as nothing else is stated, the following **assumptions** and **conditions** apply:

- A linear elastic-perfectly plastic *material behaviour* as shown in Figure 1.11 is assumed.
- In terms of the beam theory, occurring deformations are small. For that reason, geometric correlations may be linearised.
- The cross section shape of a beam is sustained when exposed to loads and deformations.
- For biaxial bending with axial force, *Bernoulli's* hypothesis is assumed, which states that the cross sections remain plane and that the influence of the shear stresses on the deformations due to shear forces is neglected (beams with infinite shear stiffness).
- For warping torsion, *Wagner's* hypothesis is assumed and the influence of the shear stresses on the rotation due to the secondary torsional moment is neglected.

1.6 Fundamental Relationships

Displacements (linear beam theory)

As is common for beams, y and z are the principal axes of the cross section and ω is the standardised warping ordinate – see Chapter 2. The longitudinal displacement u_S refers to the centre of gravity S and the displacements v_M and w_M describe the displacement of the shear centre M. For the longitudinal displacement u of an arbitrary point of the cross section the following formula applies:

$$u = u_S - y \cdot \varphi_z + z \cdot \varphi_y - \omega \cdot \psi \tag{1.1}$$

The first component is the displacement due to an axial force load. The second and the third components result from the bending moments and describe the displacements as a consequence of cross section rotations φ_y and φ_z. Here Formula (1.1) only covers displacements for which the cross section remains plane. The fourth component comprises the longitudinal displacement due to torsional loads depending on the derivative of the angle of twist ψ.

The displacements v und w in the cross section plane result from the displacement of the shear centre M and from additional components deriving from the rotation ϑ about the longitudinal axis (twist):

$$v = v_M - (z - z_M) \cdot \vartheta \tag{1.2}$$

$$w = w_M + (y - y_M) \cdot \vartheta \tag{1.3}$$

Strains

The strains are linked to the displacements by geometric relationships. According to [25], the following relations are valid for the linear beam theory. For the displacements, Formulas (1.1) to (1.3) are considered and in addition, by neglecting secondary shear deformations, it is $v'_M = \varphi_z$, $w'_M = -\varphi_y$ and $\psi = \vartheta'$.

$$\varepsilon_x = \frac{\partial u}{\partial x} = u'_S - y \cdot \varphi'_z + z \cdot \varphi'_y - \omega \cdot \vartheta'' \tag{1.4a}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = 0, \qquad \varepsilon_z = \frac{\partial w}{\partial z} = 0 \tag{1.4b, c}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \left[-(z - z_M) - \frac{\partial \omega}{\partial y} \right] \cdot \vartheta' \tag{1.4d}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \left[(y - y_M) - \frac{\partial \omega}{\partial z} \right] \cdot \vartheta' \tag{1.4e}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = -\vartheta + \vartheta = 0 \tag{1.4f}$$

Constitutive equations and stresses

The *constitutive equations* describe the correlation between stresses and strains. Neglecting the transverse strain, with the use of *Hooke's* law, a material law describing isotropic, linear elastic *material behaviour*, and the strains defined in Formulas (1.4), the following stresses can be stated:

$$\sigma_x = E \cdot \varepsilon_x = E \cdot \left(u'_S - y \cdot \varphi'_z + z \cdot \varphi'_y - \omega \cdot \vartheta'' \right) \tag{1.5}$$

$$\tau_{xy} = G \cdot \gamma_{xy} = G \cdot \left[-(z - z_M) - \frac{\partial \omega}{\partial y} \right] \cdot \vartheta' \tag{1.6}$$

$$\tau_{xz} = G \cdot \gamma_{xz} = G \cdot \left[(y - y_M) - \frac{\partial \omega}{\partial z} \right] \cdot \vartheta' \tag{1.7}$$

Internal forces and moments

Stresses can be summarised to resulting internal forces and moments. It must be pointed out that the axial force and the bending moments act at the centre of gravity, while shear forces, the torsional moments as well as the warping bimoment are related to the shear centre – see Figure 1.8.

Table 1.2 Internal forces and moments as resultants of stresses

Condition	Internal force/moment	Definition
$\sum F_x = 0:$	axial force	$N = \int_A \sigma_x \cdot dA$
$\sum V_y = 0:$	shear force	$V_y = \int_A \tau_{xy} \cdot dA$
$\sum V_z = 0:$	shear force	$V_z = \int_A \tau_{xz} \cdot dA$
$\sum M_x = 0:$	torsional moment	$M_x = \int_A \left[\tau_{xz} \cdot (y - y_M) - \tau_{xy} \cdot (z - z_M)\right] \cdot dA$ $M_x = M_{xp} + M_{xs}$
$\sum M_y = 0:$	bending moment	$M_y = \int_A \sigma_x \cdot z \cdot dA$
$\sum M_z = 0:$	bending moment	$M_z = -\int_A \sigma_x \cdot y \cdot dA$
	warping bimoment	$M_\omega = \int_A \sigma_x \cdot \omega \cdot dA$

Division of linear beam theory (infinite shear stiffness) into four subproblems

Table 1.3 shows four subproblems – biaxial bending with axial force and torsion – associated with the linear theory of beams with infinite shear stiffness. The table contains an allocation of loads, displacements and internal forces/moments as well as information concerning the equilibrium of a beam element and the stress σ_x.

Table 1.3 Division of the linear beam theory according to [25]

	"Axial force"	"Bending about the z-axis"	"Bending about the y-axis"	"Torsion"
Loads	q_x; F_x	q_y; F_y; M_{zL}	q_z; F_z; M_{yL}	m_x; M_{xL}; $M_{\omega L}$
Deformations	$u = u_S$	$v = v_M$ $u = -y \cdot v'_M$	$w = w_M$ $u = -z \cdot w'_M$	ϑ $u = -\omega \cdot \vartheta'$ $v = -(z - z_M) \cdot \vartheta$ $w = (y - y_M) \cdot \vartheta$
Internal forces and moments	N	M_z V_y	M_y V_z	M_ω $M_x = M_{xp} + M_{xs}$
Equilibrium	$N' = -q_x$	$M'_z = -V_y$ $V'_y = -q_y$	$M'_y = V_z$ $V'_z = -q_z$	$M'_\omega = M_{xs}$ $M'_x = -m_x$
$\sigma_x =$	$\frac{N}{A}$ $= E \cdot u'_S$	$-\frac{M_z}{I_z} \cdot y$ $= -E \cdot y \cdot v''_M$	$\frac{M_y}{I_y} \cdot z$ $= -E \cdot z \cdot w''_M$	$\frac{M_\omega}{I_\omega} \cdot \omega$ $= -E \cdot \omega \cdot \vartheta''$

1.7 Limit States and Load Combinations

Limit states

The limit states of structures to be analysed and the corresponding load combinations are defined in “load standards” such as DIN 1055 [7] and EC 1 [9]. For the application additional information is given in the standards (e.g. DIN 18800 [8], EC 3 [10]). In this context, the bearing capacity of a structure characterises the ability of the carrying members to resist all loadings which may occur during the erection work and the service life. The **ultimate limit state** describes a load situation of the structure where a violation of the limit would lead to a calculative collapse or a comparable failure, for example a rupture or a loss of stability and stable equilibrium, respectively. The demands on the ultimate limit state are related to the safety of people and the safety of the building including its equipment and facilities. In general, the states which may have to be observed cover the loss of the position stability (lifting, overturning, buoying upwards), the failure of the structure or its members including the foundation (rupture, changeover in a kinematic chain, loss of stability) and the failure due to fatigue influences on the material and other time-related effects. With regard to steel structures, the ultimate limit state to be verified depends on the verification method (see Table 1.1):

- beginning of a plastification
- cross section being fully plasticised at one position
- formation of a kinematic chain
- rupture

Other limit states that may be relevant are: flexural buckling, lateral torsional buckling, plate and shell buckling as well as fatigue. In general, it has to be verified, for the entire structure and its members, that the design value of the internal forces and moments or stresses S_d due to the design loading F_d is smaller than the design resistance R_d:

$$S_d \leq R_d \tag{1.8}$$

The **servicability limit state** describes the conditions of a building beyond which it can no longer be used for its designated purpose. The demands on the serviceability are related to the function of the building, the safety of people and the structural appearance. It has to be verified that the design value of stress at the serviceability limit state does not exceed the design value of a serviceability criterion (e.g. tolerable deformations). Limit states for the serviceability are not specified in DIN 18800 and they are usually arranged and agreed on individually if they are not specified in other basic or engineering standards.

Since the ultimate limit state is the basis of a safe design, ensuring that the structure and its parts do not fail, is primary focus of this book.

Design loads and resistances

The safety concepts of the German and European standards are very similar. Both use so-called partial safety factors γ_F and γ_M for the determination of the design loads and resistances. These factors increase the "actual" loads to the design level and decrease the resistances accordingly. The factor γ_F considers a possible unfavourable deviation of the load in terms of the statistical spatiotemporal spread and, in addition, possible insecurities in the mechanical and stochastic model. The factor γ_M includes the spread of the particular resistance value and also covers inaccuracies in the mechanical model related to the calculation of the resistances.

The design value of a load F_d is determined by:

$$F_d = \gamma_F \cdot \psi \cdot F_k \tag{1.9}$$

Here, γ_F is the partial safety factor which is associated with the particular load and F_k is the characteristic value of the load. If necessary, a combination factor ψ as stated in Eq. (1.9) may be considered.

The design value of the resistance parameters M_d is calculated by dividing the characteristic value of the resistance M_k (e.g. strength of the material $f_{y,k}$ and $f_{u,k}$) with the partial safety factor γ_M:

$$M_d = M_k / \gamma_M \tag{1.10}$$

Load combinations and resistance at the ultimate limit state

For the verification of the bearing capacity of a steel structure at the ultimate limit state different load combinations have to be examined which are mainly classified as follows:

- basic combinations
- exceptional combinations

For the **basic combinations** two separate cases with corresponding loads F have to be considered. According to DIN 18800, this results in the following combinations:

- permanent (dead) loads G and **all** variable loads Q_i acting unfavourably:

$$\gamma_{F,G} \cdot G_k \oplus \sum_{i>1} \gamma_{F,Q} \cdot \psi_i \cdot Q_{i,k} \tag{1.11a}$$

- permanent (dead) loads G and **one** unfavourable variable load Q_i at a time:

$$\gamma_{F,G} \cdot G_k \oplus \gamma_{F,Q} \cdot Q_{i,k} \tag{1.11b}$$

To clarify that the loads in the combination are rather combined and not necessarily directly added to each other, possibly due to acting in different directions or even at different positions of the structure, the symbol "$\oplus$" is used.

The design value of the permanent loads G_d is determined by:

$$G_d = \gamma_F \cdot G_k \qquad \text{with} \quad \gamma_F = \gamma_{F,G} = 1.35 \tag{1.12}$$

If the permanent load reduces the stress due to the variable loads, the partial safety for the permanent load has to be set to γ_F = 1.0. It should be mentioned that additional rules are specified in the standards concerning the reduction of stress due to parts of the permanent loads.

The design value of the variable loads $Q_{i,d}$ of the combinations with one unfavourable variable load at a time is

$$Q_d = \gamma_F \cdot Q_{i,k} \qquad \text{with} \quad \gamma_F = \gamma_{F,Q} = 1.5 \tag{1.13a}$$

and for all variable loads acting unfavourably it is:

$$Q_d = \gamma_F \cdot \psi_i \cdot Q_{i,k} \qquad \text{with} \quad \gamma_F = \gamma_{F,Q} = 1.5 \text{ and } \psi_i = 0.9 \tag{1.13b}$$

For **exceptional combinations**, design values of the permanent loads G_d, all variable loads $Q_{i,d}$ and one exceptional load $F_{A,d}$ have to be considered. In contrast to Formulas (1.12) and (1.13b), the partial safety factor is used with γ_F = 1.0 here. The design value for the exceptional load $F_{A,d}$ is determined with a partial factor of γ_F = 1.0 as well.

At the ultimate limit state, the partial safety factor for the resistance is usually taken with:

$$\gamma_M = 1.1 \tag{1.14}$$

The factor is not only used for the determination of the design material strength but has to be used for the design stiffness as well, which is determined with the nominal values of the cross section properties and the characteristic values of the elasticity modulus or the shear modulus, respectively. If the stability of members is not decisive, the factor γ_M may be taken as 1.0.

Load combinations and resistance at the serviceability limit state

The safety factors γ_F, combination factors ψ and load combinations to be considered for the verification have to be arranged individually if they are not specified in different basic or engineering standards. At the serviceability limit state a partial safety factor of γ_M = 1.0 is usually valid.

1.8 Introductory Example

The following example is aimed to give a first overview of the verification methods according to DIN 18800 given in Table 1.1. In doing so, the main focus is set to the ultimate limit state. Due to the significance of this state as the basis of a safe design, as mentioned previously, it is the main focus of this book. Figure 1.12 illustrates a two-span girder with a uniformly distributed load to be verified. The distributed load is considered to consist of two components, one due to the dead load and one component including the snow loads, as shown in the figure.

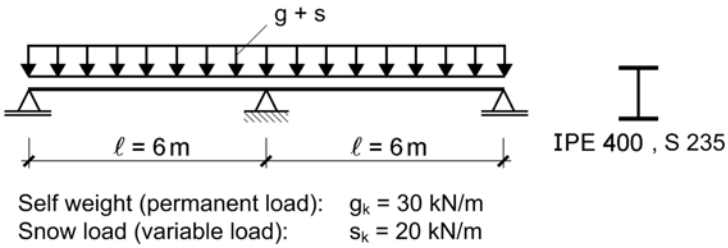


Figure 1.12 Structural system of the introductory example

The calculation of the design load values follows with the load combination of Eq. (1.11b) regarding the partial safety factors γ_F = 1.35 for the permanent load and γ_F = 1.50 for **one** variable load according to Eq. (1.12) and (1.13a). This leads to the following design load q_d:

$$q_d = g_d + s_d = 1.35 \cdot 30 + 1.5 \cdot 20 = 40.5 + 30 = 70.5 \text{ kN/m}$$

With the partial safety factor of γ_M = 1.1, the design yield strength of steel S 235 is:

$$f_{y,d} = 24.0/1.1 = 21.82 \text{ kN/cm}^2$$

Verification method Elastic-Elastic

First of all, the stress in the system is determined by calculating the internal forces and moments. The mid support plays a key role for the verification of the bearing capacity since here the internal forces and moments are at maximum (see Figure 1.13). Using the internal forces and moments, maximum stresses can be calculated, leading to the following verifications:

$$\tau_m = \frac{V}{A_{web}} = \frac{264.38}{33.2} = 7.96 \text{ kN/cm}^2 < \frac{21.82}{\sqrt{3}} = 12.6 \text{ kN/cm}^2$$

$$\max \sigma = \frac{M}{W} = \frac{31\,725}{1160} = 27.35 \text{ kN/cm}^2 > 21.82 \text{ kN/cm}^2$$

$\Rightarrow$ verification is not successful!

The (necessary) verification of the equivalent stress

$$\sigma_v = \sqrt{\sigma^2 + 3 \cdot \tau^2} \le f_{y,d}$$

cannot be successful due to max $\sigma > f_{y,d}$.

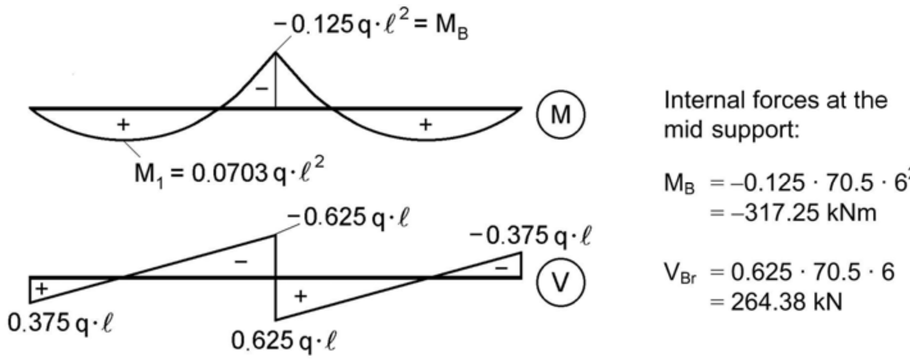


Figure 1.13 Bending moment and shear force according to the elastic theory

Verification method Elastic-Plastic

In order to verify the system in Figure 1.12, the plastic capacities of the cross section bearing capacity can be taken into consideration. Using the Elastic-Plastic procedure, the internal forces and moments are calculated according to the elastic theory – see Figure 1.13. For the verification of a sufficient load-bearing capacity the interaction conditions (e.g. DIN 18800) or the partial internal forces method can now be applied (see Chapter 8).

The use of the interaction conditions according to DIN 18800 requires knowledge of the internal forces and moments at the perfectly plastic state. By using the profile tables [29], $M_{pl,d} = 285.2$ kNm and $V_{pl,d} = 419$ kN can directly be obtained. This leads to the following verification:

$$\frac{V}{V_{pl,d}} = \frac{264.38}{419} = 0.631 > 0.33 \text{ and } < 0.9$$

$$\Rightarrow \quad 0.88 \cdot \frac{M}{M_{pl,d}} + 0.37 \cdot \frac{V}{V_{pl,d}}$$

$$= 0.88 \cdot \frac{317.25}{285.2} + 0.37 \cdot 0.631 = 0.979 + 0.234 = 1.21 > 1$$

$\Rightarrow$ verification is not successful!

Verification method Plastic-Plastic

As shown with the previous verification, it is not possible to verify the bearing capacity of the system in Figure 1.12 if the plastic reserves of the cross section are regarded at one position of the beam, which is, in this case, at the mid support. However, after the bearing capacity is reached at that position, a plastic hinge will develop and the system will still be able to carry additional loads since it will not be kinematic at that load stage. With the development of the plastic hinge (cross section in a perfectly plastic state) at the mid support due to M_B and V_B, the interaction condition used with the Elastic-Plastic procedure has to be fulfilled exactly ("= 1" instead of "≤ 1"). With $V/V_{pl,d} > 0.33$, it is:

$$0.88 \cdot \frac{M_B}{M_{pl,d}} + 0.37 \cdot \frac{V_B}{V_{pl,d}} = 1 \Rightarrow M_B = \frac{M_{pl,d}}{0.88} \cdot \left(1 - 0.37 \cdot \frac{V_B}{V_{pl,d}}\right) = 324 - 0.287 \cdot V_B$$

This formula describes what maximum bending moment the cross section is able to carry at B with regard to the acting shear force.

Figure 1.14 illustrates the structural system regarding the symmetry after the formation of the plastic hinge. For reasons of clarity, the subscript "d" to point out the design loads is neglected here. With regard to the equilibrium of the beam, the following formulas can be stated for the internal forces depending on the position x:

$$V(x) = \frac{q \cdot \ell}{2} - \frac{M_B}{\ell} - q \cdot x \qquad \Rightarrow \qquad |V_B| = \frac{q \cdot \ell}{2} + \frac{M_B}{\ell}$$

$$M(x) = \frac{q \cdot \ell}{2} \cdot x - \frac{M_B}{\ell} \cdot x - \frac{q \cdot x^2}{2}$$

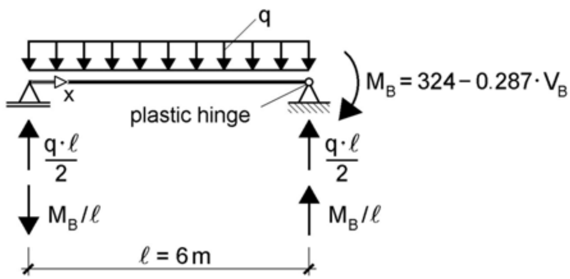


Figure 1.14 Structural system after insertion of a plastic hinge at the mid support

With the equilibrium, the shear force at the support V_B can be determined in terms of M_B, as shown above. By regarding this relationship in the previous equation for M_B, which was gained from the interaction condition, a formula for the calculation of the moment can be stated, which is now independent of V_B:

$$M_B = 324 - 0.287 \cdot V_B \text{ and } V_B = \frac{q \cdot \ell}{2} + \frac{M_B}{\ell}$$

$$\Rightarrow M_B = 324 - 0.287 \cdot \left(\frac{q \cdot \ell}{2} + \frac{M_B}{\ell} \right)$$

$$\Rightarrow M_B = \frac{324 - 60.7}{1.0478} = 251.3 \text{ kNm}$$

The formation of the plastic hinge at the mid support, i.e. the full plastification, corresponds to M_B = 251.3 kNm. This moment is smaller than $M_{pl,d}$ = 285.2 kN due to the action of the shear force. It now has to be checked whether the arising internal forces and moments within the beam span can be carried by the cross section. The decisive stress in the span is caused by the internal bending moment. It reaches its maximum at the position V(x) = 0. Using the equilibrium formulation for V(x), this leads to:

$$V(x) = \frac{q \cdot \ell}{2} - \frac{M_B}{\ell} - q \cdot x = 0 \Rightarrow x = \frac{\ell}{2} - \frac{M_B}{q \cdot \ell} = 2.404 \text{ m}$$

At that position, the bending moment can now be calculated with the equilibrium equation of M(x):

$$\max M_F = \frac{70.5 \cdot 6}{2} \cdot 2.404 - \frac{251.3}{6} \cdot 2.404 - \frac{70.5 \cdot 2.404^2}{2}$$
$$= 508.4 - 100.7 - 203.7 = 204 \text{ kNm}$$

For the verification within the beam span the internal forces and moments are considered with V = 0 and max M_F = 204 kNm, leading to the following condition:

$$\frac{M}{M_{pl,d}} = \frac{204}{285.2} = 0.72 < 1$$

If the condition is fulfilled, there is no development of a plastic hinge within the beam span. Therefore, the system will not form a plastic mechanism (chain) as failure mode and the load-bearing capacity can be verified using the Plastic-Plastic method. However, it should be mentioned that additional verifications are necessary:

- local buckling of cross section parts and sufficient cross section rotation capacity with existing b/t ≤ limit b/t (conditions are fulfilled for an IPE 400)
- lateral torsional buckling if the deformations v and ϑ are not sufficiently restricted (by bracings for instance)
- load transmission of support reactions into the beam; where required, stiffeners may have to be installed
- if necessary, verifications at the serviceability limit state

1.9 Content and Outline

Figure 1.15 contains an overview of the chapters of this book showing their interrelationship. The aim of the figure is to show which chapters are based on one another. At the same time, it gives information about which basic knowledge is of advantage for the understanding of a given chapter.

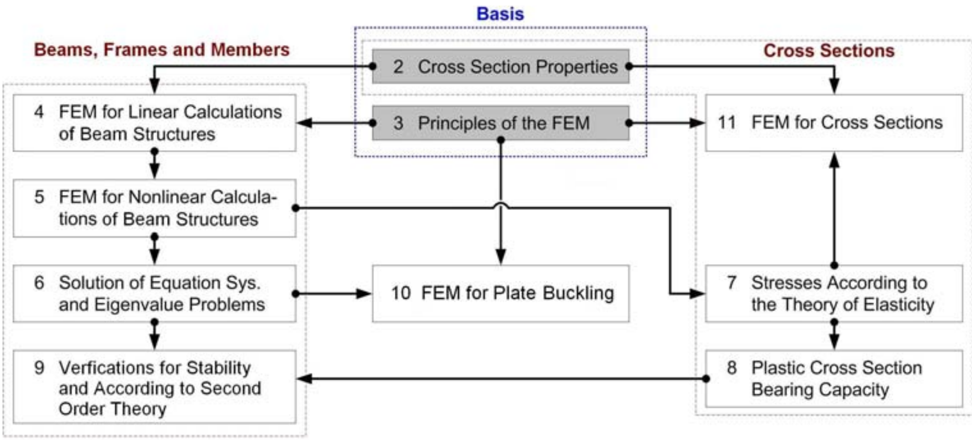


Figure 1.15 Chapter structure and dependencies

As shown in Figure 1.15, Chapters 2 and 3 are of foundational character. In **Chapter 2** the cross section properties arising in beam theory are discussed. Their knowledge is of fundamental importance for the application of beam theory (Chapters 4, 5, 6 and 9) and for a further treatment of cross sectional issues (Chapters 7, 8, 11). **Chapter 3** gives information about the principles of the finite element method (FEM). The basic idea of the method is needed for the understanding of Chapters 4, 5, 10 and 11 dealing with the numerical approach for beams and frameworks, for plates and for cross sections of beams.

Chapters 4, 5, 6 and 9 deal exclusively with the topic “beams, frames and members”. Here, the numerical backgrounds and procedures, the solution methods and the verification of bearing capacity are dealt with in detail. Since beams have a special importance in steel construction, these chapters are a central part of the book. With regard to the formulation of finite beam elements, the cross section properties (Chapters 2 and 11) are of significance and for the verification of members the resistance of the cross sections (Chapter 7 and 8).

Figure 1.15 shows that Chapters 2, 7, 8 and 11 can be described by the umbrella term “cross sections”. While in **Chapter 2** the cross section properties arising from beam theory are discussed, **Chapters 7** and **8** give information about the bearing capacity